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Localisation in weakly coupled disordered chains

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Abstract. A renormalisation group method for calculating the localisation length of coupled disordered chains is described. A system consisting of two chains with weak interchain coupling is studied numerically using the method. The results indicate that the localisation length has a strong dependence on the interchain coupling. The localisation length is generally increased by weak interchain coupling if the two chains have about the same disorder strength, while it decreases with increasing interchain coupling if the difference between the disorder strengths of the two chains is large.

1. Introduction

One-dimensional (1D) disordered systems are of particular interest in connection with the study of localisation because some rigorous results are available in these systems. The interest in 1D systems has also increased remarkably with the synthesis and thorough investigation of a wide class of quasi-1D materials over the last decade. The most important result for 1D systems is that all the eigenstates are localised for any disorder (Mott and Twose 1961, Borland 1963, Ishii 1973). However, we should keep in mind that there is no rigorous 1D material. Interchain coupling exists in all quasi-1D systems, although it may be very weak. It is now well known that the weak transverse coupling plays an important role in phase transition problems (see Firsov *et al* 1985). However, the weak interchain coupling is ignored in most of the theoretical works on localisation. The problem of how far the weak interchain coupling affects the localisation in quasi-1D disordered systems is still unsolved.

In recent years, both analytical works (Pendry and Castano 1988, Markos 1988) and numerical methods (Pichard and Sarma 1981, Mackinnon and Kramer 1981, Zheng 1986) have been developed to study the localisation in coupled disordered chains. In this paper we extend the renormalisation group (RG) method proposed by Robbins and Koiller (1985) for calculating the localisation lengths of a single isolated disordered chain to multichain systems. Using this method we have calculated the localisation lengths of a two-chain system with weak interchain coupling. Our numerical results show that, although the weak interchain coupling does not change the fact that all eigenstates are localised for arbitrarily small non-zero disorder, it can have various effects on the

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localisation length. For the case when the two chains have about the same disorder strengths, the weak interchain coupling generally increases the localisation length. However, the localisation length decreases with increasing interchain coupling when the difference between the disorder strengths of the two chains is large.

2. Method

Since Goncalves da Silva and Koiller (1981) introduced the RG method to study the spectra of mass disordered chains, the method has been successfully used in studying the densities of states of 1D disordered systems (Koiller *et al* 1983, Robbins and Koiller 1983, Langlois *et al* 1983, Liu *et al* 1984, Makler *et al* 1985, Hwang *et al* 1986, Tan and Yang 1988). In most of these works, the local Green functions are obtained by asserting that the rescaling process commutes with the configuration average. The RG method has also been extended to study the densities of states of a Bethe lattice (D'Albuquerque e Castro 1984), Husimi cacti (Anda *et al* 1984), coupled chains (Liu and Chao 1986) and a square lattice (Yang and Tan 1989). Similar methods have been used to study incommensurate systems (Jose 1983, Wiecko and Roman 1984). Robbins and Koiller (1985) have derived a formula for obtaining the localisation length of a single chain from the RG recursion relation. The formula is exact if the rescaling process is carried out for a long chain with no configuration average. Here we will follow a similar line to Robbins and Koiller (1985) to investigate the localisation length of coupled chains using the RG technique with no configuration average.

We consider an infinite strip consisting of M chains, defined by the Hamiltonian

$$H = \sum_{I=-\infty}^{\infty} (H(I) + H(I, I+1) + H(I, I-1)) \tag{1}$$

where

$$H(I) = \sum_{i=1}^{M} \varepsilon_i(I)|i, I\rangle\langle i, I| + \sum_{i=1}^{M-1} V_{i,i+1}(I, I)|i, I\rangle\langle i+1, I| + \sum_{i=2}^{M} V_{i,i-1}(I, I)|i, I\rangle\langle i-1, I| \tag{2}$$

$$H(I, I \pm 1) = \sum_{i=1}^{M} V_{i,i}(I, I \pm 1)|i, I\rangle\langle i, I \pm 1|. \tag{3}$$

The equation of motion for the Green function

$$G(Z - H) = 1 \qquad (Z = E + \mathrm{i}0^+) \tag{4}$$

can be expressed in the matrix form

$$(Z - H(I))G(I, J) = \delta(I, J) + H(I, I-1)G(I-1, J) + H(I, I+1)G(I+1, J) \tag{5}$$

where $G(I, J)$ is a $M \times M$ matrix defined as

$$G_{i,j}(I, J) = \langle i, I|G|J, J\rangle. \tag{6}$$

Equation (5) has exactly the same form as that for a single chain obtained by Koiller *et al* (1983). Therefore, it is easy to get the RG recursion relation by using the cluster

decimation technique (Liu *et al* 1984, Makler *et al* 1985, Liu and Chao 1986, Yang and Tan 1989)

$$(Z - H^{(k)}(I))G^{(k)}(I,J) = \delta(I,J) + H^{(k)}(I,I-1)G^{(k)}(I-1,J) + H^{(k)}(I,I+1)G^{(k)}(I+1,J) \qquad I = -\infty,\ldots,-1,0,1,\ldots,\infty \tag{7}$$

with

$$H^{(k+1)}(I) = H^{(k)}(2I) + H^{(k)}(2I,2I-1)(Z - H^{(k)}(2I-1))^{-1}H^{(k)}(2I-1,2I) + H^{(k)}(2I,2I+1)(Z - H^{(k)}(2I+1))^{-1}H^{(k)}(2I+1,2I) \tag{8}$$

$$H^{(k+1)}(I,I+1) = H^{(k)}(2I,2I+1)(Z - H^{(k)}(2I+1))^{-1}H^{(k)}(2I+1,2I+2) \tag{9}$$

$$G^{(k+1)}(I,J) = G^{(k)}(2I,2J) \tag{10}$$

where the superscript k denotes the number of renormalisations. Using equations (7) and (10) we can get

$$G(2^k,0) = G^{(k)}(1,0) = (Z - H^{(k)}(1))^{-1}H^{(k)}(1,0)G^{(k)}(0,0) + (Z - H^{(k)}(1))^{-1}H^{(k)}(1,2)G^{(k)}(2,1). \tag{11}$$

As k increases, $H^{(k)}(1,0)$ and $H^{(k)}(1,2)$ tend to zero at about the same rate, and $G^{(k)}(2,1)$ also tends to zero. $G^{(k)}(0,0)$ is generally a non-zero matrix. Hence, for enough large k the last term in equation (11) is negligible and then we have

$$G(2^k,0) = (Z - H^{(k)}(1))^{-1}H^{(k)}(1,0)G^{(k)}(0,0) \qquad \text{for large } k. \tag{12}$$

The localisation length of the coupled chains can be defined as (Economou 1983)

$$L = \lim_{k\to\infty} 2N \ln\left(\sum_{\substack{i=1\\j=1}}^{M} G_{i,j}(N,0)G^{*}_{i,j}(N,0)\right)^{-1}. \tag{13}$$

Substituting equation (12) into equation (13) and considering the fact that, at the fixed point, $H^{(k)}(1,0)\to 0$ while $(Z - H^{(k)}(1))^{-1}$ and $G^{(k)}(0,0)$ are finite constant matrices except for a set of measure zero where $\det(Z - H^{(\infty)}(I)) = 0$ (see figure 1), the localisation length can be expressed in the form

$$L = \lim_{k\to\infty} 2^{k+1}\left(\ln\sum_{\substack{i=1\\j=1}}^{M} H^{(k)}_{i,j}(1,0)H^{*(k)}_{i,j}(1,0)\right)^{-1}. \tag{14}$$

3. Numerical results

In order to study the effect of weak interchain coupling on the localisation of quasi-1D disordered systems, we have calculated the localisation lengths of a system consisting of two chains by using the RG method. We believe the two-chain system is a good approximation of a quasi-1D system in the weak interchain coupling limit.

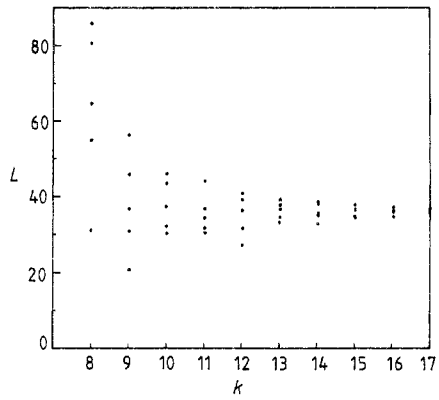

Figure 1. The convergence of the localisation length at $E = 0$ of a two-chain system with $W1 = W2 = 2$ and $U = 0.1$ calculated from equation (14), for five samples.

We assume that the system is pure diagonal disordered. The distribution of the site energy is described by a distribution function

$$P(\varepsilon_i(I)) = \begin{cases} 1/W1 & |\varepsilon_i(I)| < W1/2 \quad \text{for } I = 1 \\ 1/W2 & |\varepsilon_i(I)| < W2/2 \quad \text{for } I = 2. \end{cases} \tag{15}$$

The nearest neighbour intrachain hopping is

$$V_{i,i+1}(I, I) = V_{i+1,i}(I, I) = 1 \tag{16}$$

and the interchain coupling is

$$V_{1,2}(I, I) = V_{2,1}(I, I) = U. \tag{17}$$

Before presenting the numerical results, we should point out that, although equation (14) is exact only in the limit $k \to \infty$, the practical calculations are performed with finite k values. The localisation lengths calculated for different samples are widely dispersed when k is small. However, the dispersion of the results becomes weaker and weaker as k increases, as shown in figure 1. Therefore, a well defined localisation length can be obtained with enough large k. In our calculation, we have taken $2^k \sim 1000L$.

We have first considered the case when the two chains have the same disorder strength, i.e. $W1 = W2 = 2$. Figure 2 shows the localisation length of the two-chain system against energy for interchain coupling strength $U = 0.1$, compared with the results for a single isolated chain with disorder strength $W = 2$. Since the localisation length is symmetric in energy about $E = 0$, only results for positive energies are shown. It can be seen that the localisation length of the coupled chains is greater than that of the single isolated chain within the whole band. We can also see that there appear two peaks for the values of localisation length of the two-chain system near the band centre $E = 0$. This is different from the single-chain case, for which there appears only one peak at $E = 0$.

Figure 3 shows the interchain coupling dependence of the localisation length of a two-chain system with $W1 = W2 = 2$ calculated for different energies. As is seen from the figure, in the weak coupling case ($U \leqslant 0.1$) the localisation length increases drastically with increasing interchain coupling strength for all calculated energies. This suggests that the localisation length of the quasi-1D disordered system is very sensitive to weak interchain coupling. However, after reaching a maximum, the localisation length

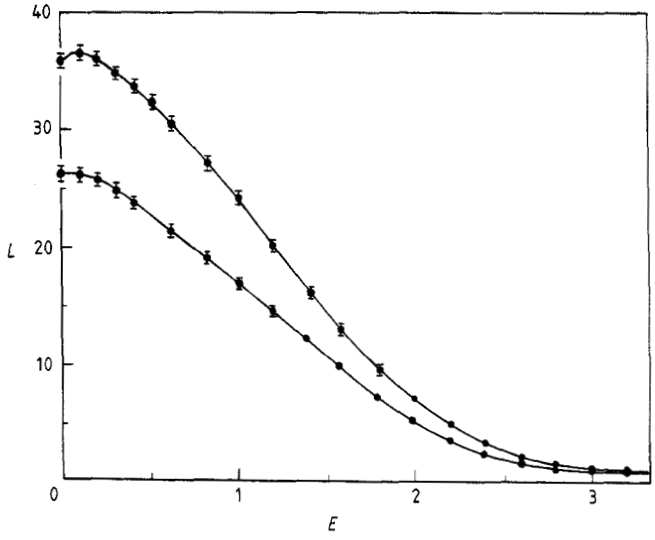

Figure 2. The localisation length as a function of energy for a two-chain system with $W1 = W2 = 2$ and $U = 0.1$ (upper curve), compared with that for a single chain with disorder strength $W = 2$ (lower curve).

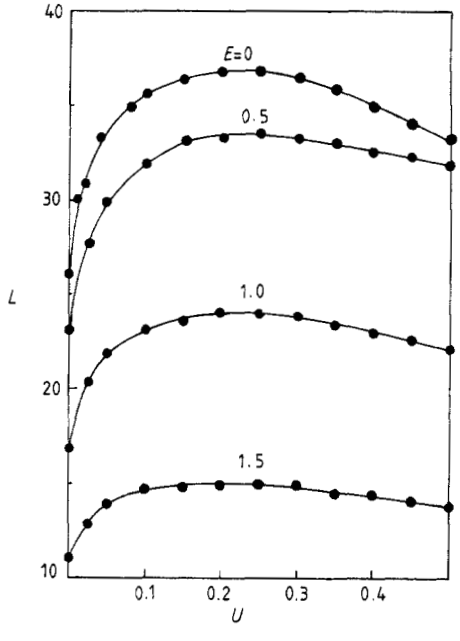

Figure 3. The interchain coupling dependence of the localisation length for a two-chain system with $W1 = W2 = 2$.

decreases with increasing coupling strength. It is easy to understand this phenomenon if we consider the following facts: the two-chain system is a good approximation of the quasi-1D system only for the very weak interchain coupling case. The energy band of the two-chain system becomes wider and wider as the interchain coupling strength is increased. It can split into two bands symmetric about $E = 0$ for large enough interchain coupling and the energy range near $E = 0$ may lie in the band gap. Hence, we think this

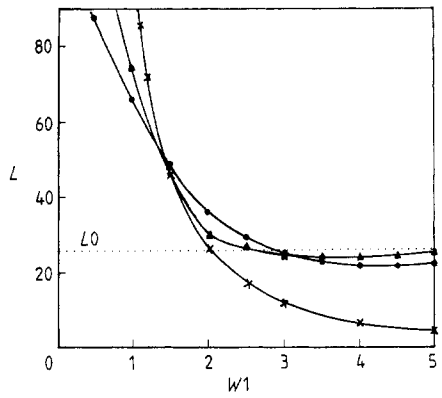

Figure 4. The localisation length at $E = 0$ of a two-chain system with $W1 = W2 = 2$ plotted against $W1$, for $U = 0.1$ (●) and 0.01 (▲). The localisation length at $E = 0$ for a single isolated chain with disorder strength $W1$ is also plotted for comparison (×). $L0$, indicated by the dotted line, is the localisation length of a single chain with disorder strength $W = 2$ at $E = 0$.

decrease is a two-chain effect which does not occur in systems consisting of an infinite number of chains.

We have also examined a system consisting of two chains with different disorder strengths, i.e. $W1 \neq W2$. Taking $W2 = 2$, we have calculated the localisation lengths of the system for various values of $W1$ and U. The results are shown in figure 4. It is seen that the weak interchain coupling increases the localisation length only in the parameter region $W1 \ll W2$. For the cases $W1 \ll W2$ and $W2 \ll W1$, however, the localisation length decreases with increasing interchain coupling. The localisation length is not sensitive to the coupling strength in the case $W1 \gg W2$. As $W1 \to \infty$, obviously, the localisation length of the system converges to a constant $L0$, regardless of the interchain coupling strength. $L0$ equals the localisation length of a single chain with disorder strength $W = 2$ at $E = 0$. We can also see in figure 4 that the effect of the interchain coupling is very important in the case $W1 \ll W2$. This importance can be seen more clearly by considering the transport property of a two-chain system with $W1 = 0$ and $W2 \neq 0$. If there is no interchain coupling, the two-chain system has a finite conductance because chain 1 provides a non-scattering channel for the electrons (Anderson *et al* 1980). However, if interchain coupling exists, arbitrary non-zero small interchain coupling can introduce the random scattering of chain 2 into chain 1 and causes all states in the system to be localised. Hence, the conductance of the system tends to zero as the length of the sample goes to infinity.

Another interesting problem is how the interchain coupling affects the localisation length of a quasi-1D disordered system if the interchain coupling matrix element is also a random variable. In order to give a preliminary answer to the question we have considered a two-chain system with $W1 = W2 = 2$, and $V_{1,2}(I, I) = V_{2,1}(I, I) = U(I)$, where $U(I)$ is a random variable described by a distribution function

$$P(U(I)) = 1/U0 \qquad 0 \leqslant U(I) \leqslant U0. \tag{18}$$

We have calculated the $U0$ dependence of the localisation length of the system and the results are shown in figure 5. We can see that the results shown in figure 5 are similar to those shown in figure 3. In the weak interchain coupling case, the localisation length of the two-chain system increases with increasing interchain coupling strength $U0$. In the strong coupling case, however, the localisation length decreases with increasing $U0$. The most important message we can get from figure 5 is that the weak interchain coupling

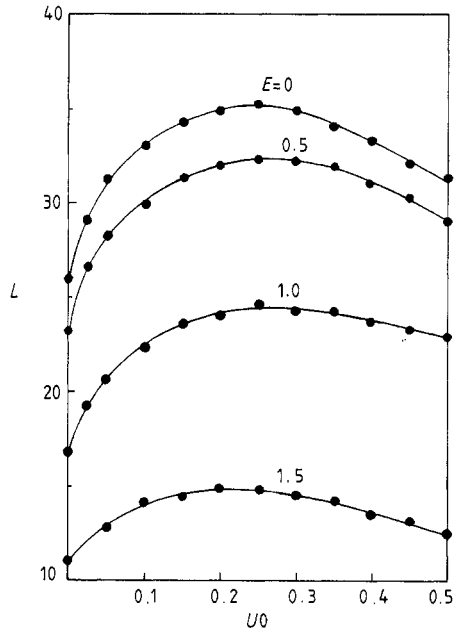

Figure 5. The $U0$ dependence of the localisation length for a two-chain system with $W1 = W2 = 2$.

generally increases the localisation length of the quasi-1D disordered system even if the interchain hopping matrix element is a random variable.

4. Conclusion

We have presented a renormalisation group (RG) method for calculating the localisation lengths of coupled chains. The method is applicable for systems with various kinds of disorder. In order to investigate the effects of weak interchain coupling on the localisation of electrons in quasi-1D disordered systems, we have calculated the localisation lengths of a two-chain system using the RG method. Our results are mostly for the weak interchain coupling limit. For the case in which the two chains have the same disordered strength, i.e. $W1 = W2$, the localisation length is drastically increased by the weak interchain coupling, even if the interchain hopping matrix element is a random variable. The case in which the two chains have different disorder strengths, i.e. $W1 \neq W2$, provides a more interesting effect of interchain coupling. It is found that the localisation length increases with increasing weak interchain coupling only in the parameter region $W1 \simeq W2$ whereas the localisation length decreases as the interchain coupling increases in the parameter regions $W1 \gg W2$ and $W1 \ll W2$.

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